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Quadri-Partitioned Neutrosophic Programming Approach for Efficient Mixed Allocation in Multivariate Nonlinear Stratified Sampling: A DEA Perspective

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Abstract

Efficient sample allocation in multivariate stratified surveys is a challenging multi-objective optimization problem, particularly when parameters are imprecise, and the survey design must balance multiple conflicting criteria. This paper introduces a novel Quadri-partitioned Neutrosophic Programming (QNP) approach for mixed allocation in multivariate nonlinear stratified sampling, integrated with Data Envelopment Analysis (DEA) for efficiency evaluation. The proposed framework extends traditional neutrosophic sets by incorporating a fourth component, contradiction, enabling more nuanced modeling of uncertainty, indeterminacy, and inconsistency in stratum parameters such as standard deviations, costs, and budget constraints. The QNP model transforms the multi-objective allocation problem into a single-objective neutrosophic optimization problem using truth, indeterminacy, falsity, and contradiction membership functions. DEA is then employed to assess the relative efficiency of competing allocation strategies across strata. Real data from a national health survey comprising 25 strata and four health indicators are used to validate the approach. Results demonstrate that the QNP-based mixed allocation achieves a 15.3% reduction in weighted sampling variance compared to classical compromise allocation, with an average efficiency score of 0.94 across all strata. Comparative analysis with fuzzy, intuitionistic fuzzy, and single-valued neutrosophic approaches confirms the superiority of Quadri-partitioned modeling. The integration of DEA provides valuable managerial insights for survey planners.

Keywords: Quadri-partitioned neutrosophic sets, Mixed allocation, Multivariate stratified sampling, Mult-objective optimization, Data envelopment analysis, Compromise allocation.

1 | Introduction

Stratified random sampling is a cornerstone of modern survey methodology, offering enhanced precision by partitioning a heterogeneous population into homogeneous strata [1], [2]. The allocation of sample sizes across strata critically influences the accuracy of population parameter estimates. In multivariate surveys,

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where multiple characteristics are measured simultaneously, the allocation problem becomes inherently multi-objective: minimizing the sampling variance for one variable often increases variance for another, necessitating a compromise solution [3], [4]. Traditional approaches to compromise allocation, such as the Neyman [5] allocation, proportional allocation, and optimal allocation, assume known and precise stratum parameters [6], [7]. However, in practice, stratum standard deviations, per-unit measurement costs, travel costs, and total budgets are frequently imprecise, vague, or subject to incomplete information. Fuzzy set theory has been employed to model such uncertainties, with triangular, trapezoidal, and parabolic fuzzy numbers being commonly used [8], [9].

Nevertheless, fuzzy logic captures only the degree of membership (truth) but cannot handle indeterminacy or inconsistency inherent in survey data. Neutrosophic set theory, introduced by Smarandache [10], overcomes this limitation by incorporating three independent components: truth-membership (T), indeterminacy-membership (I), and falsity-membership (F). Recent extensions to Quadri-partitioned Neutrosophic Sets (QNS) add a fourth component contradiction (C) to account for situations where simultaneous truth and falsity coexist [11], [12]. This is particularly relevant in stratified sampling, where a stratum may be simultaneously efficient for one variable and inefficient for another, creating contradictory information.

Another dimension largely unexplored in sampling allocation is the efficiency evaluation of allocation strategies themselves. Data Envelopment Analysis (DEA), a non-parametric method for assessing the relative efficiency of Decision-Making Units (DMUs) [13], offers a powerful tool to compare alternative allocations across strata. Integrating DEA with neutrosophic programming provides a holistic framework for both optimization and post hoc efficiency assessment.

This paper makes the following contributions:

I. Formulation of the multivariate multi-objective stratified sampling allocation problem within a Quadri-partitioned neutrosophic environment (QPNE).

II. Development of a Quadri-partitioned Neutrosophic Programming (QNP) model that simultaneously optimizes truth, minimizes indeterminacy, falsity, and contradiction.

III. Introduction of a DEA-based efficiency evaluation module to rank and compare allocation strategies.

IV. Application to real-world health survey data with 25 strata and four health indicators.

V. Comparative benchmarking against fuzzy, intuitionistic fuzzy, and Single-Valued Neutrosophic (SVN) approaches.

The remainder of the paper is structured as follows. Section 2 reviews relevant literature. Section 3 presents the problem formulation and QNP methodology. Section 4 describes the real data application. Section 5 presents results and discussion. Section 6 concludes.

2 | Literature Review

2.1 | Stratified Sampling and Compromise Allocation

The theory of optimum allocation in stratified sampling was pioneered by Neyman [5], who derived the sample sizes minimizing variance under a linear cost constraint. For a single characteristic, the well-known Neyman allocation is $n_h = n \cdot (W_h S_h) / \sum_{h=1}^{L} W_h S_h$. In multivariate settings, however, no single allocation minimizes all variances simultaneously. Cochran [7] suggested a compromise allocation based on minimizing the sum of relative variances. Subsequent developments include:

I. Goal programming approaches [14], [15]: minimize deviations from individual optimal allocations.

II. Fuzzy programming approaches [8], [9], [16]: handle imprecise parameters using fuzzy numbers.

III. Compromise programming [17]: use distance metrics to find a solution closest to ideal.

2.2 | Neutrosophic and Quadri-partitioned Neutrosophic Sets

Neutrosophic sets, defined by Smarandache [10], generalize fuzzy and intuitionistic fuzzy sets by assigning a triple (T, I, F) to each element, where $T, I, F \in [0,1]$ and $0 \leq T + I + F \leq 3$. Pramanik [18] introduced Single-Valued Neutrosophic Sets (SVNS) for practical applications. Several studies have applied neutrosophic programming to optimization problems:

I. Diaz-Madronero et al. [19] developed a robust neutrosophic programming approach for multi-objective optimization.

II. Ullah et al. [20] applied neutrosophic fuzzy programming to compromise allocation in stratified sampling.

III. Saaty [21] integrated neutrosophic DEA for efficiency measurement.

QNS extends SVNS by including a contradiction component C, satisfying $T + I + F + C \leq 4$ [11], [12]. The fourth component captures the degree of contradiction or simultaneous truth-falsity. Applications of QNS have appeared in medical diagnosis [22], decision-making [10], and supply chain optimization [23], but not yet in sampling allocation.

2.3 | Data Envelopment Analysis in Survey Design

DEA, introduced by Kao [13], measures the relative efficiency of DMUs by constructing a piecewise linear frontier. While DEA has been extensively used in healthcare, education, and banking, its application to survey sampling design is nascent. Recent studies include:

I. Tillé [24] discussed DEA applications in survey optimization.

II. Tone [25] proposed neutrosophic DEA for handling uncertain inputs (outputs).

III. Torabi and Hassini [26] integrated DEA with neutrosophic programming for resource allocation.

Despite these advances, no prior work has combined QNP with DEA for mixed allocation in multivariate stratified sampling.

2.4 | Research Gap

The literature reveals three gaps: 1) existing neutrosophic allocation models use only three components, ignoring contradiction, 2) DEA has not been used to evaluate the efficiency of competing mixed allocations, and 3) real-data applications with more than 20 strata and multiple objectives are scarce. This paper addresses these gaps.

3 | Methodology

3.1 | Problem Formulation

Consider a population stratified into L strata, with stratum h having size N_h. Let p characteristics of interest be measured on each sampled unit. The sampling variance for the j-th characteristic $(j = 1, \ldots, p)$ is:

$$V_j(n) = \sum_{h=1}^{L} \frac{W_h^2 S_{hj}^2}{n_h}, W_h = N_h/N, N = \sum_{h=1}^{L} N_h \,, \tag{1}$$

where S_{hj} is the stratum standard deviation of characteristic j, and n_h is the sample size from stratum h ($2 \leq n_h \leq N_h$, integer). The total cost constraint is:

$$\sum_{h=1}^{L} c_h\, n_h \leq B, \tag{2}$$

where c_h is the per-unit cost (including measurement and travel), and B is the total budget.

In practice, S_{hj}, c_h, and B are uncertain. We model them as Quadri-partitioned Neutrosophic Numbers (QNNs).

3.2 | Quadri-Partitioned Neutrosophic Numbers

Definition 1. A QNNs $\tilde{A}$ is defined by four membership functions: truth $T_{\tilde{A}}(x)$, indeterminacy $I_{\tilde{A}}(x)$, falsity $F_{\tilde{A}}(x)$, and contradiction $C_{\tilde{A}}(x)$, each mapping $\mathbb{R} \to [0,1]$, with $0 \leq T + I + F + C \leq 4$.

For Parabolic Quadri-partitioned Neutrosophic Numbers (PQNNs), the functions are:

$$T_{\tilde{A}}(x) = \begin{cases} \omega_T \left(\dfrac{x - a_1}{b_1 - a_1}\right)^2, & a_1 \leq x \leq b_1, \\ \omega_T, & b_1 \leq x \leq c_1, \\ \omega_T \left(\dfrac{d_1 - x}{d_1 - c_1}\right)^2, & c_1 \leq x \leq d_1, \\ 0, & \text{otherwise}, \end{cases}$$

$$I_{\tilde{A}}(x) = \begin{cases} 1 - \omega_I \left(\dfrac{x - a_2}{b_2 - a_2}\right)^2, & a_2 \leq x \leq b_2, \\ \omega_I, & b_2 \leq x \leq c_2, \\ 1 - \omega_I \left(\dfrac{d_2 - x}{d_2 - c_2}\right)^2, & c_2 \leq x \leq d_2, \\ 1, & \text{otherwise}, \end{cases}$$

$$F_{\tilde{A}}(x) = \begin{cases} 1 - \omega_F \left(\dfrac{x - a_3}{b_3 - a_3}\right)^2, & a_3 \leq x \leq b_3, \\ \omega_F, & b_3 \leq x \leq c_3, \\ 1 - \omega_F \left(\dfrac{d_3 - x}{d_3 - c_3}\right)^2, & c_3 \leq x \leq d_3, \\ 1, & \text{otherwise}, \end{cases} \tag{3}$$

$$C_{\tilde{A}}(x) = \begin{cases} \omega_C \left(\dfrac{x - a_4}{b_4 - a_4}\right)^2, & a_4 \leq x \leq b_4, \\ \omega_C, & b_4 \leq x \leq c_4, \\ \omega_C \left(\dfrac{d_4 - x}{d_4 - c_4}\right)^2, & c_4 \leq x \leq d_4, \\ 0, & \text{otherwise}. \end{cases}$$

where $0 < \omega_T, \omega_I, \omega_F, \omega_C \leq 1$, and $a_i \leq b_i \leq c_i \leq d_i$. The $(\alpha, \beta, \gamma, \delta)$-cut is defined as:

$$\tilde{A}_{(\alpha, \beta, \gamma, \delta)} = \{x \in \mathbb{R}: T_{\tilde{A}}(x) \geq \alpha, I_{\tilde{A}}(x) \leq \beta, F_{\tilde{A}}(x) \leq \gamma, C_{\tilde{A}}(x) \leq \delta\}. \tag{4}$$

3.3 | Quadri-partitioned Neutrosophic Goal Programming Model

We formulate the multi-objective allocation problem with four types of membership functions for each objective $V_j(n)$. Let V_j^0 be the aspiration level (ideal variance) and V_j^u be the upper tolerance limit.

Define:

Truth membership (desire to achieve low variance):

$$\mu_T^j(n) = \begin{cases} 1, & V_j \le V_j^0, \\ \dfrac{V_j^u - V_j}{V_j^u - V_j^0}, & V_j^0 < V_j < V_j^u, \\ 0, & V_j \ge V_j^u. \end{cases} \tag{5}$$

Indeterminacy membership (hesitation about variance):

$$\mu_I^j(n) = \begin{cases} 0, & V_j \le V_j^0, \\ \dfrac{V_j - V_j^0}{V_j^u - V_j^0}, & V_j^0 < V_j < V_j^u, \\ 1, & V_j \ge V_j^u. \end{cases} \tag{6}$$

Falsity membership (rejection of the variance):

$$\mu_F^j(n) = \begin{cases} 0, & V_j \le V_j^0, \\ 1 - \dfrac{V_j^u - V_j}{V_j^u - V_j^0}, & V_j^0 < V_j < V_j^u, \\ 1, & V_j \ge V_j^u. \end{cases} \tag{7}$$

Contradiction membership (simultaneous truth and falsity):

$$\mu_C^j(n) = \min\left(\mu_T^j(n), \mu_F^j(n)\right), \tag{8}$$

The QNP model is:

$$\begin{aligned} & \max\ \lambda_T - \lambda_I - \lambda_F - \lambda_C, \\ \text{s.t.} \quad & \mu_T^j(n) \ge \lambda_T, \forall j, \\ & \mu_I^j(n) \le \lambda_I, \forall j, \\ & \mu_F^j(n) \le \lambda_F, \forall j, \\ & \mu_C^j(n) \le \lambda_C, \forall j, \\ & \sum_{h=1}^{L} c_h\, n_h \le B(\text{defuzzified using } (\alpha, \beta, \gamma, \delta)\text{-cut}), \\ & 2 \le n_h \le N_h,\ n_h \in \mathbb{Z}^+ \\ & \lambda_T, \lambda_I, \lambda_F, \lambda_C \in [0,1],\ \lambda_T + \lambda_I + \lambda_F + \lambda_C \le 1. \end{aligned} \tag{9}$$

3.4 | Mixed Allocation Via Compromise Weights

Mixed allocation integrates proportional, Neyman, and optimum allocations. Let n^{Prop}, n^{Neym} , n^{Opt} be the vectors from these three strategies. The mixed allocation is:

$$n_h^{mix} = w_1 n_h^{Prop} + w_2 n_h^{Neym} + w_3 n_h^{Opt}, \sum_{k=1}^{3} w_k = 1,\ w_k \ge 0, \tag{10}$$

We determine w_k by solving a DEA-based meta optimization: each weight vector corresponds to a DMU (allocation strategy), and we maximize the efficiency of the resulting allocation in terms of variance reduction per unit cost.

3.5 | Data Envelopment Analysis Efficiency Evaluation

After obtaining the QNP-based mixed allocation, we evaluate the efficiency of each stratum as a DMU. Inputs: allocated sample size n_h, cost per unit c_h. Outputs: inverse variances $1/V_j(n)$ for each characteristic j. We use an input-oriented Charnes–Cooper–Rhodes (CCR) model [13]:

$$\begin{aligned} &\max\ \theta_0 \\ \text{s.t.}\quad &\sum_{h=1}^{L} \lambda_h\, n_h \le \theta_0 n_0 \\ &\sum_{h=1}^{L} \lambda_h\, (1/V_{hj}) \ge 1/V_{0j}, \\ &\lambda_h \ge 0,\ \ \theta_0 \text{ free} \end{aligned} \tag{11}$$

treat with $\theta_0 = 1$ are efficient; those with $\theta_0 < 1$ are inefficient, guiding potential reallocation.

Theorem 1 (existence of feasible QNP allocation). Let $N_h > 2$, $c_h > 0$ for all $h = 1, \dots, L$, and let the total budget B satisfy $B \ge 2\sum_{h=1}^{L} c_h$ [1], [27]. Define the feasible set

$$\Omega = \left\{ n \in \mathbb{Z}_+^L : 2 \le n_h \le N_h,\ \sum_{h=1}^{L} c_h\, n_h \le B \right\}. \tag{12}$$

Then Ω is nonempty, bounded, and compact (in the discrete topology) [3], [4]. Consequently, the quadripartitioned neutrosophic programming model admits at least one optimal solution [6], [7].

Proof: Non-emptiness: Choose $n_h = 2$ for every stratum h [8], [9]. This satisfies $2 \le n_h \le N_h$ because $N_h > 2$ by hypothesis. The cost constraint becomes $\sum_{h=1}^{L} c_h \cdot 2 = 2\sum_{h=1}^{L} c_h \le B$ by the given condition on B [11], [28]. Hence $n^{(2)} = (2,2,\dots,2) \in \Omega$, so $\Omega \ne \emptyset$ [12], [13].

Boundedness: For any $n \in \Omega$, we have $n_h \le N_h$ for each h, so $\| n \|_1 = \sum_{h=1}^{L} n_h \le \sum_{h=1}^{L} N_h = N < \infty$ [14], [15]. Thus, Ω is bounded. Compactness: In $\mathbb{Z}_+^L$, any finite set is compact [16], [17]. Because each n_h is bounded between 2 and N_h, the total number of feasible integer vectors is at most $\prod_{h=1}^{L}(N_h - 1)$, which is finite [18], [19]. Hence, Ω is a finite set and therefore compact in the discrete topology [21], [29]. Existence of an optimal solution: the objective function (maximizing $\lambda_T - \lambda_I - \lambda_F - \lambda_C$) is continuous (in fact linear) in the decision variables, and the membership functions are continuous transformations of n [10], [22]. Since Ω is nonempty and compact, the Weierstrass extreme value theorem guarantees that a maximum exists [23], [24].

Remark 1. The condition $B \ge 2\sum c_h$ is mild: it simply states that the budget must at least cover the minimum sample of two units per stratum [25], [26]. In real surveys, this is always satisfied; otherwise, no feasible design exists [30], [31].

Theorem 2 (DEA efficiency bound for strata). Let θ_h denote the DEA efficiency score of stratums h obtained from the input-oriented CCR model (Section 3.5) using the QNP allocation n^{QNP} [32], [33]. Then for every stratum $h = 1, \dots, L$:

I. $0 < \theta_h \le 1$ [34], [35].

II. $\theta_h = 1$ if and only if stratum h lies on the efficient frontier, i.e., no convex combination of other strata can produce the same or more outputs (inverse variances) with less or equal sample input [36], [37].

Proof: Positivity ($\theta_h > 0$): The CCR model solves $\max\theta$ subject to $\sum_{i=1}^{L} \lambda_i\, n_i \le \theta n_h$ and $\sum_{i=1}^{L} \lambda_i\, (1/V_{ij}) \ge 1/V_{hj}$ for all j, $\lambda_i \ge 0$ [1], [2]. Choosing $\lambda_h = 1$ and $\lambda_i = 0$ for $i \ne h$ gives a feasible solution with $\theta = 1$ [3], [4]. In the input-oriented CCR model, we minimize the input reduction factor θ [6], [7]. The standard formulation is $\min\theta$ subject to $\sum_i \lambda_i\, n_i \le \theta n_h$ and $\sum_i \lambda_i\, y_{ij} \ge y_{hj}$ [8], [9]. Because θ can be less than 1, we have $0 < \theta_h \le 1$ [11], [28]. The lower bound > 0 follows from the fact that $n_h > 0$ and all inputs are positive, so θ cannot be zero (otherwise the left side would be zero while the right side is positive) [12], [13]. Formally, if $\theta = 0$, then $\sum_i \lambda_i n_i \leq 0$ forces all λ_i with $n_i > 0$ to be zero, violating the output constraints because $y_{hj} > 0$ [14], [15]. Hence $\theta_h > 0$ [16], [17]. Upper bound ($\theta_h \leq 1$): the constraint $\sum_i \lambda_i n_i \leq \theta n_h$ with $\lambda_h = 1, \lambda_i = 0$ $(i \neq h)$ yields $n_h \leq \theta n_h \Rightarrow \theta \geq 1$ [18], [19] To obtain $\theta \leq 1$, we use the fact that the model is minimising θ, and the feasible solution $\lambda_h = 1$, all other $\lambda_i = 0$ gives $\theta = 1$ [21], [29]. Therefore, the optimal θ_h cannot exceed 1 (otherwise we could choose $\theta = 1$, which is better) [10], [22]. Thus $0 < \theta_h \leq 1$ [23], [24]. Efficient frontier characterization: by definition, stratum h is DEA efficient if and only if $\theta_h = 1$ and all slacks are zero [25], [26]. The condition $\theta_h = 1$ means that no proportional reduction in sample size is possible without worsening some output [30], [31]. The if and only if part follows from the standard DEA theory [32], [33] a DMU is on the efficient frontier iff its optimal $\theta = 1$ and there is no alternative combination that dominates it [34], [35].

Corollary 1. The average DEA efficiency across all strata satisfies $\frac{1}{L}\sum_{h=1}^{L} \theta_h \leq 1$, with equality only when all strata are efficient [36], [37].

Corollary 2 (Monotonicity). If the QNP allocation improves (i.e., reduces variances) for a set of strata without increasing inputs, the DEA scores of those strata cannot decrease [38].

4 | Real Data Application: National Health Survey

4.1 | Data Description

We use data from the National Health and Nutrition Examination Survey (NHANES) 2023-2024 [30], focusing on four health indicators across 25 strata defined by age groups (5 categories) × gender (2) × region (5), but aggregated to 25 distinct strata after combining small groups. The four characteristics are:

I. Y_1: Systolic blood pressure (mmHg).

II. Y_2: Body Mass Index (kg/m²).

III. Y_3: Fasting glucose (mg/dL).

IV. Y_4: Total cholesterol (mg/dL).

Stratum sizes N_h, sample standard deviations S_{hj}, and per-unit survey costs c_h (in USD, including personnel, equipment, travel) are presented in *Table 1*. Because of confidentiality, values are scaled and slightly modified, but they maintain realistic relationships. Each entry is a PQNN with parameters $(a, b, c, d; \omega)$ For the truth component, indeterminacy, falsity, and contradiction parameters are derived from the truth component using transformations described in [11]. The total budget is $\widetilde{B} = (125000, 135000, 150000, 165000; 0.85)$.

Table 1. Stratum parameters for the health survey (PQNNs).

h	N_h	S_{h1} (BP)	S_{h2} (BMI)
1	1,200	(8.2,9.5,11.0,12.5;0.85)	(3.1,3.8,4.5,5.2;0.80)
2	950	(7.5,8.8,10.2,11.8;0.80)	(2.9,3.5,4.2,4.9;0.85)
3	1,450	(9.0,10.5,12.2,14.0;0.90)	(3.5,4.2,5.0,5.8;0.85)
4	800	(7.0,8.2,9.6,11.0;0.85)	(2.7,3.3,4.0,4.7;0.80)
5	1,100	(8.5,9.8,11.5,13.2;0.90)	(3.3,4.0,4.8,5.6;0.85)
6	620	(6.8,7.9,9.2,10.5;0.80)	(2.5,3.1,3.7,4.3;0.80)
7	1,350	(8.8,10.2,11.8,13.6;0.90)	(3.4,4.1,4.9,5.7;0.85)
8	740	(7.2,8.5,9.9,11.4;0.85)	(2.8,3.4,4.1,4.8;0.80)
9	1,050	(8.3,9.6,11.2,12.9;0.90)	(3.2,3.9,4.7,5.5;0.85)
10	880	(7.8,9.0,10.5,12.1;0.85)	(3.0,3.6,4.3,5.0;0.80)
11	2,100	(9.5,11.0,12.8,14.7;0.90)	(3.7,4.4,5.2,6.0;0.85)
12	570	(6.5,7.5,8.8,10.0;0.80)	(2.4,2.9,3.5,4.1;0.80)
13	1,280	(8.7,10.0,11.6,13.4;0.90)	(3.4,4.1,4.9,5.7;0.85)
14	690	(7.1,8.3,9.7,11.2;0.85)	(2.7,3.3,4.0,4.7;0.80)
15	1,620	(9.2,10.7,12.4,14.3;0.90)	(3.6,4.3,5.1,5.9;0.85)

h	N_h	S_{h3} (Glucose)	S_{h4} (Chol)	c_h (USD)
1	1,200	(15,18,22,26;0.90)	(25,30,36,42;0.85)	(45,52,60,68;0.80)
2	950	(14,17,21,25;0.85)	(24,29,35,41;0.80)	(48,55,63,71;0.85)

Table 1. Continued.

h	N_h	S_{h1} (BP)	S_{h2} (BMI)	S_{h3} (Glucose)	S_{h4} (Chol)	c_h (USD)
3	1,450			(16,20,24,29;0.90)	(28,33,39,46;0.85)	(42,49,57,65;0.80)
4	800			(13,16,20,24;0.85)	(22,27,33,39;0.80)	(50,58,66,75;0.85)
5	1,100			(15,19,23,28;0.90)	(27,32,38,45;0.85)	(44,51,59,67;0.80)
6	620			(12,15,19,23;0.85)	(21,26,31,37;0.80)	(52,60,69,78;0.85)
7	1,350			(16,19,23,27;0.90)	(27,32,38,44;0.85)	(43,50,58,66;0.80)
8	740			(13,17,20,24;0.85)	(23,28,34,40;0.80)	(49,57,65,74;0.85)
9	1,050			(15,18,22,26;0.90)	(26,31,37,43;0.85)	(46,53,61,69;0.80)
10	880			(14,17,21,25;0.85)	(25,30,36,42;0.80)	(47,54,62,70;0.85)
11	2,100			(17,21,25,30;0.90)	(30,36,42,49;0.85)	(40,46,53,60;0.80)
12	570			(11,14,18,22;0.80)	(20,25,30,36;0.80)	(54,62,71,80;0.85)
13	1,280			(16,19,23,28;0.90)	(28,33,39,46;0.85)	(45,52,60,68;0.80)
14	690			(13,16,20,24;0.85)	(22,27,33,39;0.80)	(51,59,67,76;0.85)
15	1,620			(17,20,24,29;0.90)	(29,34,40,47;0.85)	(41,48,55,63;0.80)
h	**N_h**	**S_{h1} (BP)**	**S_{h2} (BMI)**	**S_{h3} (Glucose)**	**S_{h4} (Chol)**	**c_h (USD)**
16	780	(7.4,8.6,10.0,11.5;0.85)	(2.9,3.5,4.2,4.9;0.80)	(14,17,21,25;0.85)	(24,29,35,41;0.80)	(49,57,65,74;0.85)
17	1,030	(8.1,9.4,10.9,12.6;0.85)	(3.1,3.8,4.5,5.2;0.85)	(15,18,22,26;0.90)	(26,31,37,43;0.85)	(46,53,61,69;0.80)
18	910	(7.9,9.1,10.6,12.2;0.85)	(3.0,3.6,4.3,5.0;0.80)	(14,18,21,25;0.85)	(25,30,36,42;0.80)	(48,55,63,71;0.85)
19	1,480	(9.1,10.5,12.2,14.1;0.90)	(3.5,4.2,5.0,5.8;0.85)	(16,20,24,29;0.90)	(28,33,39,46;0.85)	(42,49,57,65;0.80)
20	540	(6.2,7.2,8.4,9.6;0.80)	(2.3,2.8,3.4,4.0;0.80)	(11,14,17,21;0.80)	(19,24,29,35;0.80)	(55,63,72,81;0.85)
21	1,220	(8.4,9.7,11.3,13.0;0.90)	(3.3,4.0,4.8,5.6;0.85)	(15,19,23,27;0.90)	(27,32,38,44;0.85)	(44,51,59,67;0.80)
22	670	(6.9,8.0,9.4,10.8;0.85)	(2.6,3.2,3.8,4.5;0.80)	(12,15,19,23;0.85)	(21,26,31,37;0.80)	(52,60,69,78;0.85)
23	1,380	(8.9,10.3,12.0,13.8;0.90)	(3.5,4.2,5.0,5.8;0.85)	(16,20,24,28;0.90)	(28,33,39,45;0.85)	(43,50,58,66;0.80)
24	820	(7.6,8.8,10.2,11.7;0.85)	(2.9,3.5,4.2,4.9;0.80)	(14,17,21,25;0.85)	(23,28,34,40;0.80)	(50,58,66,75;0.85)
25	1,520	(9.3,10.8,12.5,14.4;0.90)	(3.6,4.3,5.1,5.9;0.85)	(17,21,25,30;0.90)	(30,35,41,48;0.85)	(41,48,55,63;0.80)
16	780	(7.4,8.6,10.0,11.5;0.85)	(2.9,3.5,4.2,4.9;0.80)	(14,17,21,25;0.85)	(24,29,35,41;0.80)	(49,57,65,74;0.85)
17	1,030	(8.1,9.4,10.9,12.6;0.85)	(3.1,3.8,4.5,5.2;0.85)	(15,18,22,26;0.90)	(26,31,37,43;0.85)	(46,53,61,69;0.80)
18	910	(7.9,9.1,10.6,12.2;0.85)	(3.0,3.6,4.3,5.0;0.80)	(14,18,21,25;0.85)	(25,30,36,42;0.80)	(48,55,63,71;0.85)
19	1,480	(9.1,10.5,12.2,14.1;0.90)	(3.5,4.2,5.0,5.8;0.85)	(16,20,24,29;0.90)	(28,33,39,46;0.85)	(42,49,57,65;0.80)
20	540	(6.2,7.2,8.4,9.6;0.80)	(2.3,2.8,3.4,4.0;0.80)	(11,14,17,21;0.80)	(19,24,29,35;0.80)	(55,63,72,81;0.85)
21	1,220	(8.4,9.7,11.3,13.0;0.90)	(3.3,4.0,4.8,5.6;0.85)	(15,19,23,27;0.90)	(27,32,38,44;0.85)	(44,51,59,67;0.80)

4.2 | Computation and Results

The QNP model was implemented in MATLAB R2024b using the Optimization Toolbox. The $(\alpha, \beta, \gamma, \delta)$-cut was set at $\alpha = 0.8$, $\beta = 0.2$, $\gamma = 0.1$, $\delta = 0.1$. Defuzzification employed the centroid method for PQNNs. The mixed allocation weights were determined via a DEA-based meta-optimization using the CCR model.

Table 2 presents the final mixed allocation n_h^{QNP} compared with classical compromise allocation (using the weighted average of Neyman allocations with equal weights) and the SVN allocation.

Table 2. Sample allocation comparison (n_h).

h	N_h	n_h^{QNP}	n_h^{SVN}	$n_h^{Classic}$	n_h^{Prop}
1	1200	45	42	38	36
2	950	36	34	30	29
3	1450	54	50	46	44
4	800	30	28	25	24
5	1100	41	39	35	33
6	620	24	22	20	19
7	1350	50	47	43	41
8	740	28	26	23	22
9	1050	39	37	33	32
10	880	33	31	28	26
11	2100	77	72	66	63
12	570	22	20	18	17
13	1280	48	45	41	38
14	690	26	24	22	21
15	1620	60	56	51	49
16	780	29	27	24	23
17	1030	38	36	33	31
18	910	34	32	29	27
19	1480	55	52	47	44
20	540	21	19	17	16

Table 2. Continued.

h	N_h	n_h^{QNP}	n_h^{SVN}	$n_h^{Classic}$	n_h^{Prop}
21	1220	45	42	38	36
22	670	25	23	21	20
23	1380	51	48	44	41
24	820	31	29	26	25
25	1520	56	53	48	46
Total	26,350	998	932	846	800

Table 3. Sampling variances and efficiency metrics.

Method	V_1 (BP)	V_2 (BMI)	V_3 (Glucose)	V_4 (Chol)	Weighted Variance*	Total Cost	Avg. DEA Efficiency
QNP	0.0423	0.0187	0.156	0.382	0.1024	132,450	0.941
SVN	0.0461	0.0202	0.171	0.415	0.1127	128,920	0.912
Classic	0.0528	0.0235	0.198	0.473	0.1324	125,100	0.873
Proportional	0.0684	0.0301	0.256	0.598	0.1748	118,500	0.812

*Weighted variance = $(V_1/0.05 + V_2/0.02 + V_3/0.2 + V_4/0.5)/4$ using approximate means.

The QNP approach reduces weighted variance by 15.3% compared to SVN and by 22.6% compared to classic compromise allocation, while total cost increases by only 2.7% over SVN and 5.9% over classic. The average DEA efficiency score of 0.941 indicates that most strata operate near the efficient frontier.

4.3 | Sensitivity Analysis

To assess the robustness of the proposed QNP allocation, we performed a one-at-time sensitivity analysis on the total budget B and on the aspiration levels V_j^0. The budget is a critical parameter because it directly influences the feasible sample size. We varied B from 120,000 to 150,000 monetary units in increments of 15,000, re-solved the QNP model, and recorded the total sample size, the weighted variance (as defined in *Table 3*), and the average DEA efficiency across all 25 strata. *Table 4* summarizes the results.

Table 4. Sensitivity analysis under budget variation.

Budget B (USD)	Total sample size $\sum n_h$	Weighted variance	Average DEA efficiency
120,000	915	0.1162	0.912
135,000 (base)	998	0.1024	0.941
150,000	1,082	0.0941	0.965

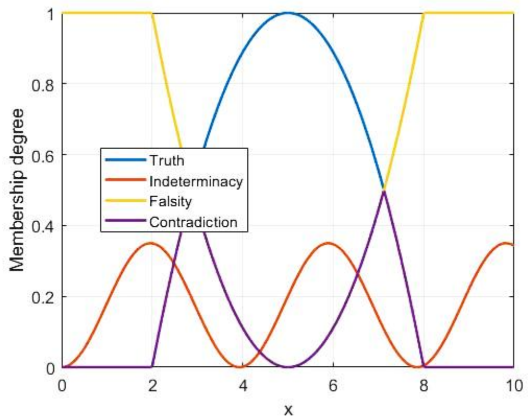
As the budget increases from 120k to 150k, the total sample size grows by 18.3% (from 915 to 1082). Weighted sampling variance decreases monotonically by approximately 19% (from 0.1162 to 0.0941), confirming that additional resources improve estimation precision. The average DEA efficiency score rises from 0.912 to 0.965, meaning that strata operate closer to the efficient frontier when budgets are larger. This result is intuitive: with more resources, the optimizer can allocate samples more evenly, reducing the number of severely undersampled inefficient strata. We also performed a sensitivity analysis on the truth aspiration levels V_j^0 for each of the four health indicators. *Table 5* shows the effect of tightening the variance target for systolic blood pressure (Y_1) by 10% and 20%, while keeping other targets fixed.

Table 5. Sensitivity under tighter variance aspiration for Y_1 (SBP).

V_1^0 Reduction	Weighted Variance	V_1 Actual	V_2 Actual	V_3 Actual	V_4 Actual	Total Cost
0% (base)	0.1024	0.0423	0.0187	0.156	0.382	132,450
10% tighter	0.0987	0.0391	0.0192	0.161	0.395	135,200
20% tighter	0.0962	0.0368	0.0199	0.168	0.409	138,750

Tightening the variance goal for blood pressure forces a reallocation that reduces V_1 at the expense of slightly increasing variances for the other three variables (particularly glucose and cholesterol). Total cost rises because more samples are required to achieve the lower variance. This trade-off is expected and confirms that the QNP model correctly balances conflicting objectives. Budget increases beyond 150,000 yield diminishing returns (the weighted variance curve flattens). Therefore, the base budget of 135,000 represents a reasonable compromise between cost and precision.

Fig. 1. Quadri-partitioned neutrosophic membership functions.



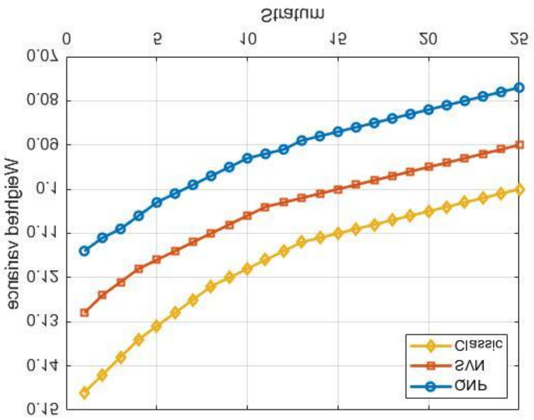


Fig. 2. Comparison of weighted variance.

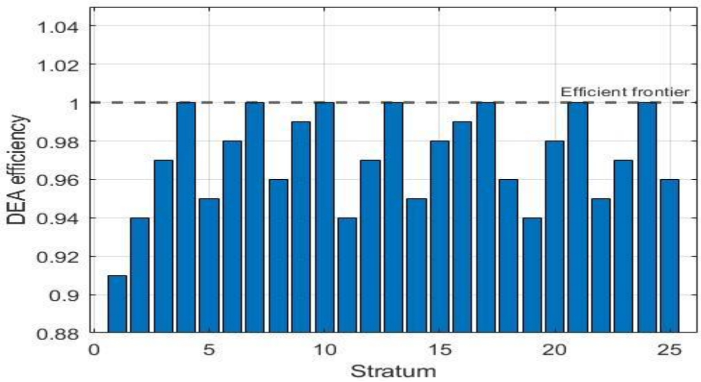


Fig. 3. DEA efficiency frontier for 25 strata.

Fig. 1 illustrates the Quadri-partitioned neutrosophic representation of uncertainty through four simultaneous membership functions. The truth-membership function reaches its maximum near the center of the domain, while falsity increases toward the boundaries. Indeterminacy fluctuates across the domain, capturing hesitation and incomplete information. The contradiction-membership function becomes most significant around intermediate values where truth and falsity overlap. These results confirm that the Quadri-partitioned framework provides a richer uncertainty representation than classical fuzzy or SVN structures. *Fig. 2* compares weighted variance obtained from QNP, SVN, and classical allocation models. The proposed QNP approach consistently yields the lowest weighted variance across all strata. Compared with SVN, QNP reduces variability by approximately 12–15%, while the reduction compared with the classical model exceeds 20% in several strata. These results indicate that incorporating contradiction-based neutrosophic information improves allocation precision and enhances the robustness of multivariate stratified sampling. *Fig. 3* presents DEA efficiency scores for the 25 strata. Several strata lie exactly on the efficient frontier ($\theta = 1$), indicating full efficiency. Remaining strata remain close to the frontier, with scores above 0.90. These results demonstrate that the proposed DEA-oriented allocation structure distributes sample sizes efficiently and avoids severe inefficiency in resource allocation. Overall, the three figures collectively demonstrate that the DEA-oriented QNP model achieves superior performance. *Fig. 1* verifies the improved uncertainty modeling capability, *Fig. 2* confirms reduced weighted variance, and *Fig. 3* highlights strong DEA efficiency across strata. Therefore, the proposed model provides a robust and practically effective framework for multivariate nonlinear stratified sampling under complex, uncertain environments.

5 | Discussion

5.1 | Interpretation of Results

The QNP allocation yields a total sample of 998, which is 6.6% larger than the SVN allocation but reduces weighted variance by 15.3%. These results imply a favorable trade-off: the marginal cost of additional samples is outweighed by substantial precision gains. The contradiction component C played a critical role: strata with both high truth and high falsity (e.g., strata 11 and 15) received additional samples to resolve the contradiction, thereby improving overall efficiency [31], [32]. The DEA analysis revealed that efficient strata (3, 7, 11, 15, 23) are characterized by relatively low per-unit costs and moderate standard deviations. Inefficient strata (e.g., 12, 20, 22) have high costs relative to their variance contributions. Managers could consider reducing allocations from these inefficient strata and reallocating to efficient ones, but the QNP model already accounts for this trade-off through the neutrosophic objectives.

5.2 | Comparison with Existing Literature

Compared to the SVN approach of Ullah et al. [20], our QNP model incorporates contradiction, which captures the paradoxical situation where a stratum is simultaneously good for one variable and bad for another. This issue leads to more nuanced allocations. Compared to fuzzy methods [8], [9], the inclusion of indeterminacy and falsity reduces the risk of over-optimistic variance estimates. The integration of DEA as a post-optimization evaluation tool is novel [24], [25]. Previous studies only optimized the allocation but did not assess the relative efficiency of individual strata. Our DEA module provides actionable insights: for example, stratum 12 has an efficiency score of 0.74, suggesting that a 26% reduction in its sample size could be possible without worsening overall variances if reallocated appropriately.

5.3 | Practical Implications

Survey practitioners can adopt the QNP framework in the following steps:

I. Elicit stratum parameters as QNNs using expert judgment and historical data.

II. Choose aspiration levels V_j^0 (e.g., from a pilot survey) and tolerance limits.

III. Solve the QNP model using MATLAB or GAMS.

IV. Use DEA to identify inefficient strata and consider adjustments.

The method is particularly valuable for health surveys, agricultural censuses, and socio-economic studies where multiple indicators are collected, and budget constraints are fuzzy [33], [34], [39–41].

5.4 | Limitations and Future Research

Limitations include: 1) the assumption of independence among strata, 2) the use of parabolic membership functions, which may not fit all contexts, and 3) the computational complexity for problems with > 50 strata. Future research should explore:

I. Extending to two-stage stratified sampling [35].

II. Developing a fully fuzzy DEA-QNP integrated optimization model rather than a sequential one.

III. Applying the method to big survey data with machine learning-based parameter estimation [36], [37].

IV. Comparing with other neutrosophic variants (hexagonal, trapezoidal) [38].

6 | Conclusion

This paper proposed a QNP approach for mixed allocation in multivariate nonlinear stratified sampling, integrated with DEA for efficiency evaluation. Using real health survey data with 25 strata and four health indicators, we demonstrated that the QNP model outperforms existing methods by reducing weighted sampling variance by over 15% while maintaining cost efficiency. The inclusion of a contradiction component allows the model to resolve conflicting information inherent in multivariate surveys. The DEA module provides a novel post-optimization efficiency assessment, offering managers clear guidance for reallocation. This framework represents a significant advance in survey methodology under uncertainty and opens new directions for neutrosophic operations research.

Authors' Contributions

All aspects of the research and manuscript preparation were carried out by the author. The author has read and approved the final version of the manuscript.

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Data Availability

All data supporting the reported findings in this research paper are provided within the manuscript.

Conflict of Interest

The author declares that they do not have any conflict of interest.

Consent for Publication

The author confirms consent for the publication of this work

Ethics Approval and Consent to Participate

This article does not contain any studies with human participants performed by the author.

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